

# Market Model Introduction

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## 2. Modelling international trade

- Major agri-food markets (>90% coverage of trade in primary agriculture)
- Processing industry: **oils, dairy, meats**
- **Global coverage with individual countries and country groups**
- Bilateral trade links, trade costs, import/export prices
- Trade policy representation
- We want to allow simultaneous export and import of goods
- Quality differences between origins, let consumers differentiate

Armington, Paul S. 1969

"A Theory of Demand for Products Distinguished by Place of Production," IMF Staff Papers 16, pp. 159-178.

# Outline

- About multi-commodity models
- Components (blocks) of the CAPRI market module
- Iterative solution
- Hands-on: Prohibitive tariffs on China's imports from the US

# What is a Multi-Commodity Model ?

- **More** than one output market, **but** not a general equilibrium
- Class of models written in **physical** and **valued** terms
- System of equations: ***no objective function***
- Same no. of endogenous variables as equations (called “square system”)
- Many examples:
  - Aglink OECD (<https://web.jrc.ec.europa.eu/policy-model-inventory/explore/models/model-aglink-cosimo/>)
  - FAPRI (<https://fapri.missouri.edu/about-fapri/>)
  - Agmemod (<https://agmemod.eu>)

# Types of MCM models

## Template models

- e.g., CAPRI market model, CGE
- **structurally identical equations** for each product and region, with market differences expressed in parameters.
- **Parameterization** -> literature research, other models
- Domestic policies in template models often use price wedges between market and producer/consumer prices
- flexible functional forms and calibration algorithms ensure parameters **align with microeconomic theory**
- Allows for a **welfare analysis of results**, as it ensures profit or utility maximization

## Econometrically estimated

- e.g., AGMEMOD, FAPRI
- Flexibility in functional relations (could be different for different products or countries)
- Parameters **rooted in observed behavior**, consistent simulation and parameter estimation.
- Parameters may **not meet micro-theory conditions**
- Not used for **welfare analysis**.

# Properties of the CAPRI market model

- Comparative-static, deterministic (stochastic), partial, spatial, global equilibrium model for **most agricultural primary** and some **secondary products**
- Including the **young animals** clearing in European regions
- **Commodities (XX)** is chosen to cover as far as possible all products used for food (O) and feed (FEED) from the **supply models**
- Excludes factor (labor and capital) markets, non-agricultural products and some agricultural products as flowers
- Spatial as it **includes bilateral trade** flows and the related **trade policy instruments**

# Exercise

Open the database of CAPRI.gdx (provided as download) with Studio and check the number of commodities in the market model using the set XX.

# Blocks of the market model in CAPRI

1. Behavioral functions (FOC)
  - Supply Side
    - Supply of primary products
    - Supply oils and cakes
    - Supply of dairy products
  - Demand Side
    - Human consumption
    - Demand for feed use
    - Demand of the processing industry
2. Armington system (Trade)
3. Balances
4. Price linkages
5. Policy representation

# 1. Behavioural functions overview

Quantity	Variable	Key equation(s)	Functional form	Driving variable(s)	Driving variable name
Supply of primary product	v_prodQuant	ProdNQ_	Normalized quadratic profit function	Producer price	v_prodPrice
Supply of cakes and oils	v_prodQuant	ProcO_	Physical relation to input	Processing yield of oilseeds	v_procYield
Supply of dairy products	v_prodQuant	DairyNQ_, ProcMargM_	Normalized quadratic profit function	Processing margin (output minus input value)	v_procMarg
Human consumption	v_consQuant	XiS_, GLDemandGiS_, GLDemandGS_, GLDemandFS_	Generalised Leontief Expenditure System	Consumer price (and DATA on income and population)	v_consPrice
Feed demand	v_feedQuant	FeedNQ_	Normalized quadratic profit function	Market feed price minus subsidies	v_arm1Price
Processing demand	v_procQuant	ProcNQ_	Profit maximization over processing margins (CET)	Market price or processing margin	v_arm1Price, v_procMarg

# Exercise

- Open market\_model.gms in gams/arm/ folder and search for the equation with the name ProdNQ\_.
- What is the role of the parameter p\_cnstNQSupp?
- What is the role of the parameter p\_hessNQSupp?

# Normalised Quadratic Profit Function

- Supply of primary and processed products represented by a **linear supply function**
- The output supply is **derived from a normalized quadratic profit function**
- We assume **profit maximization of the firms**
- The normalized quadratic profit function is widely used
- It has the advantage of being **globally convex if it is locally convex.**
- Direct solution of equations without having to rely on methods to solve a system of non-linear equations
- Processing industry NQ function and fixed processing yields

# Theory of Normalized Quadratic Profit Function

The profit function gives the **maximum** profits as a function of price  $P$ .  
 We don't (need to) know what the production function looks like,  
 but assume that profits are revenues minus costs, i.e.  $P'Q - c(Q)$  for some function  $c$

If we assume that the profit function is a quadratic one, we can write

$$\Pi(P) = A'P + \frac{1}{2}P'BP$$

Hotelling's lemma gives quantity as a function of price:

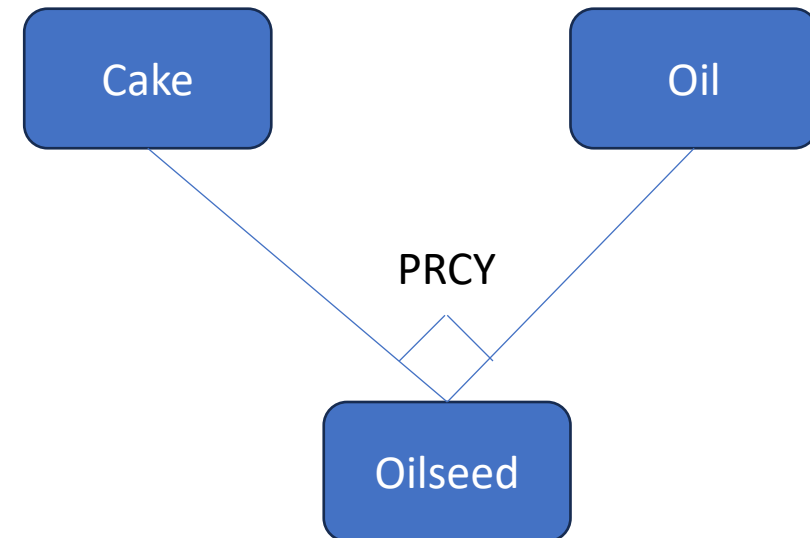
$$Q(P) = \frac{\partial \Pi}{\partial P} = A + B'P$$

To make the function independent of scale of  $P$  (currency), divide all prices by a price index  $\bar{P} = w'P$  (with  $w =$  weights).  
 That gives the "normalized quadratic profit function"

$$Q(P) = \frac{\partial \Pi}{\partial P} = A + \frac{1}{\bar{P}}B'P$$

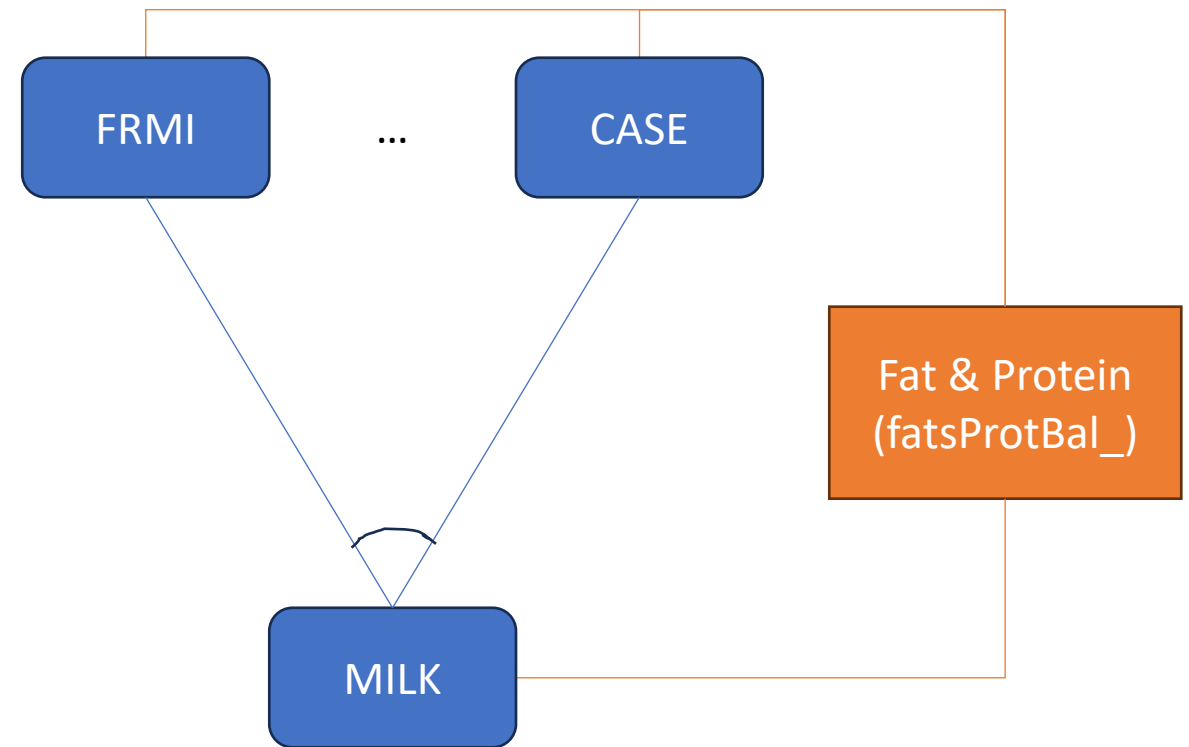
# 1. Processing of oil seeds

- Inputs:
  - Sunflower seed
  - Rapeseed
  - Soy beans
- Outputs
  - Cakes (protein meals)
  - Vegetable oils
- Behavioural drivers
  - the value of output (cakes and oils)
  - minus the value of input (oilseed)
  - at fixed processing yields



# 1. Processing of raw milk

- Inputs:
  - $MILK = COMI + SGMI$
- Outputs
  - Nine dairy products
- Behavioural drivers
  - Processing margin
  - = difference between selling price and price for milk fat and protein



# 1. Sets relating to processing

```
SET SED_TO_OIL (XX,XX) / SOYA.SOYO, SUNF.SUNO, RAPE.RAPO /;
```

```
SET SED_TO_CAK (XX,XX) / SOYA.SOYC, SUNF.SUNC, RAPE.RAPC /;
```

```
SET SED "Oilseeds" /SOYA "Soybean"  
          SUNF "Sunflower"  
          RAPE "Rapeseed"/;
```

```
SET MLK /FRMI "Fresh milk products"  
      CHES "Cheese"  
      BUTT "Butter"  
      CREM "Cream"  
      SMIP "Skimmed milk powder"  
      COCM "Concentrated milk"  
      WMIO "Whole milk powder"  
      WHEP "Whey powder"  
      CASE "Casein" /;
```

# Demand: Generalized Leontief Expenditure System

Source of derivations:

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Flexible and Semiflexible Consumer Demands with Quadratic Engel Curves

Author(s): David L. Ryan and Terence J. Wales

Reviewed work(s):

Source: *The Review of Economics and Statistics*, Vol. 81, No. 2 (May, 1999), pp. 277-287

Published by: [The MIT Press](#)

Stable URL: <http://www.jstor.org/stable/2646865>

Accessed: 29/10/2012 07:44

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# Demand continued

## Indirect utility function

(highest utility achievable at prices  $P$  at income  $Y$ )

$$v(P, Y) = \frac{-G(P)}{Y - F(P)}$$

Where  $G(P)$  and  $F(P)$  are some functions, defined later.

Roy's identity gives demand per capita,  $X_i(P, Y)$

$$X_i = \frac{-\partial v(P, Y)}{\partial P_i} / \frac{-\partial v(P, Y)}{\partial Y}$$

$$= \frac{G_i(P)}{G(P)} (Y - F(P)) + F_i(P)$$

# Demand continued

$$F = \sum_i D_i P_i$$

$$G = \sum_i \sum_j B_{i,j} \sqrt{P_i P_j}$$

Differentiate  
w.r.t.  $P_i$  ...

$$\frac{\partial F}{\partial P_i} = F_i = D_i$$

$$\frac{\partial G}{\partial P_i} = G_i = \sum_j B_{i,j} \sqrt{P_j / P_i}$$

Expenditure remaining after commitments are covered

$$\Rightarrow x_i = \left[ D_i + \frac{G_i}{G} (Y - F) \right] Pop$$

Value of minimum commitments

$D_i$  = Consumption independent of prices and income

# GLE and welfare

- Money metric (income equivalent) as the utility measurement to calculate “consumer surplus”
- Theoretically consistent
- Can be hence added/compared to costs/revenues/taxes directly to calculate overall welfare (change)
- Works as “consumer surplus”

# Final demand: GLE and welfare

Indirect utility function

$$v(P, Y) = \frac{-G(P)}{Y - F(P)}$$

Invert to expenditure function  
using  $U(X) = v(P, Y)$

$$Y \equiv e(U, P) = F(P) - \frac{G(P)}{v(P, Y)}$$

We compute: “How much income would be required at the reference prices to let the consumer reach the Utility Level obtained in the simulation?”

$$e(U_{sim}, P_{ref}) = F_{ref} - \frac{G_{ref}}{v(P_{sim}, Y_{sim})} = F_{ref} + \frac{G_{ref}}{G_{sim}} (Y_{sim} - F_{sim})$$

# Exercise

- Open the yield shock simulation in the GUI and look at the welfare table!
- Can you find the welfare numbers?

# Exercise

The screenshot shows a software interface for a 'Welfare market model'. The main window displays a table with the following data:

Region	Year	Product	Absolute diff. to Scen	cap_after_2014_refdefaultA	userScens_Yield_shock_2_6_MMOndefaultA
World	2030	All primary agricultural output			
Money metric [Mio Euro]				5326592.00	5333192.00
Agricultural profits [Mio Euro]				3460950.50	3455363.75
Profit of land owners [Mio Euro]					0.00
Profit of the feed industry [Mio Euro]					-54216.01
Profits of processing industry [Mio Euro]					847.12
Profits of dairy industry [Mio Euro]					300946.81
Tariff revenues [Mio Euro]					200.38
Domestic support outlays [Mio Euro]					0.00
Tariff rate quota rents [Mio Euro]					56664.26
Welfare [Mio Euro]					-144.68
					1866.54
					-7.96
					9078399.00
					1957.00

An overlay dialog box titled 'Transposing and Merging' is open, showing various configuration options:

- Table control area: Animation (0)
- Box (1): Region(204)
- Box (2): hide(1)
- Box (3): Year(2)
- Box (4): Product(99)
- Table area:
  - Table column groups: 0
  - Table columns: 2
  - Scen(2)
  - Table row groups: 0
  - Table rows: Welfare position(10)
  - Table cells area

## 2. Modelling international trade

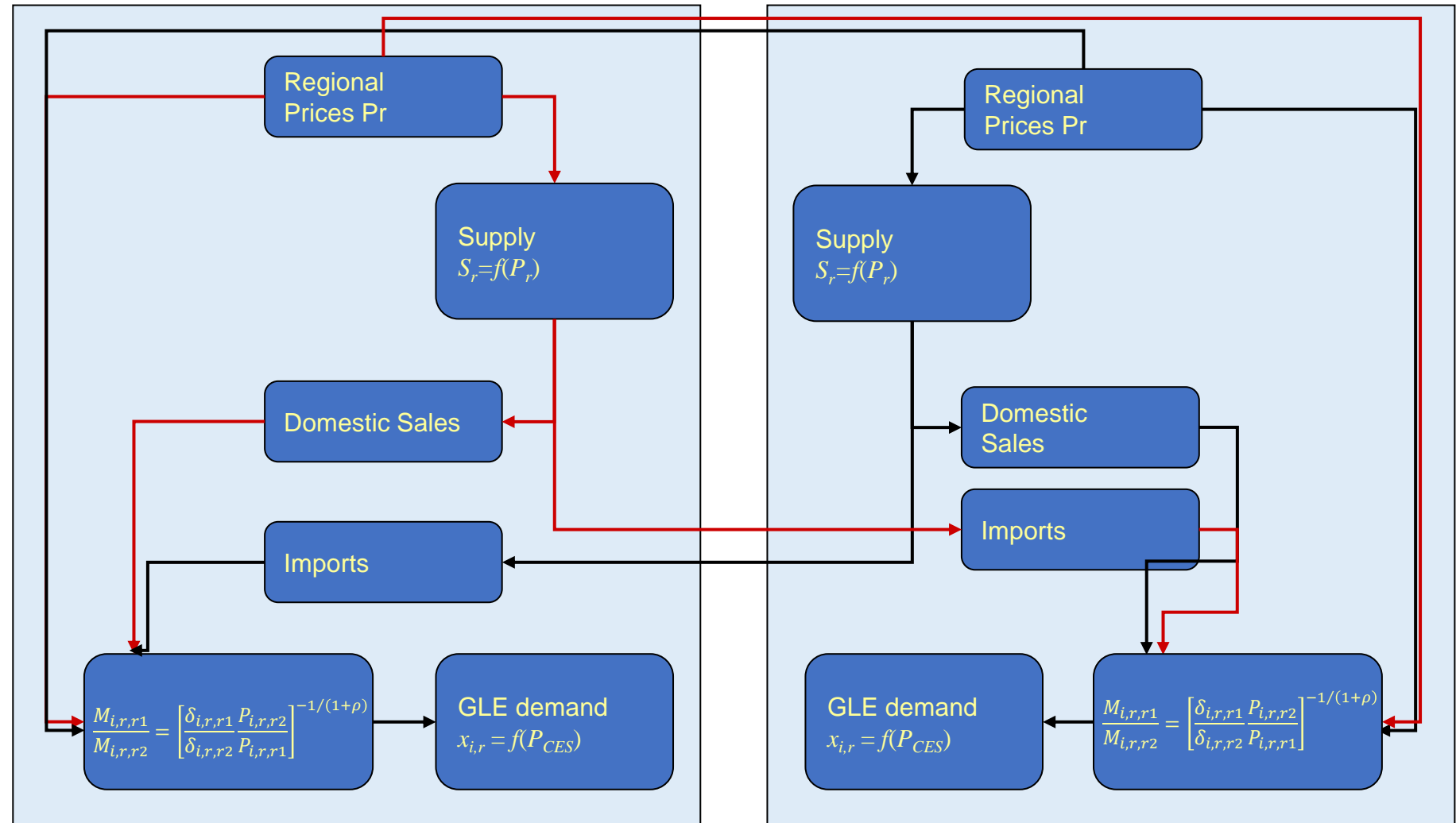
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r1

r2



**M<sub>i,r1</sub> (or M<sub>i,r2</sub>):** This likely represents the ratio of imported goods to domestically produced goods (or vice-versa) for good 'i' in a particular region .

**δ<sub>i,r1,r1</sub> and δ<sub>i,r1,r2</sub>** (and similar for r2): These are **share parameters** or **distribution parameters**. They reflect the preference or bias consumers in region 'r1' (for example) have towards goods produced in region 'r1' versus goods imported from region 'r2'. A higher δ value for a particular origin indicates a stronger preference for goods from that origin

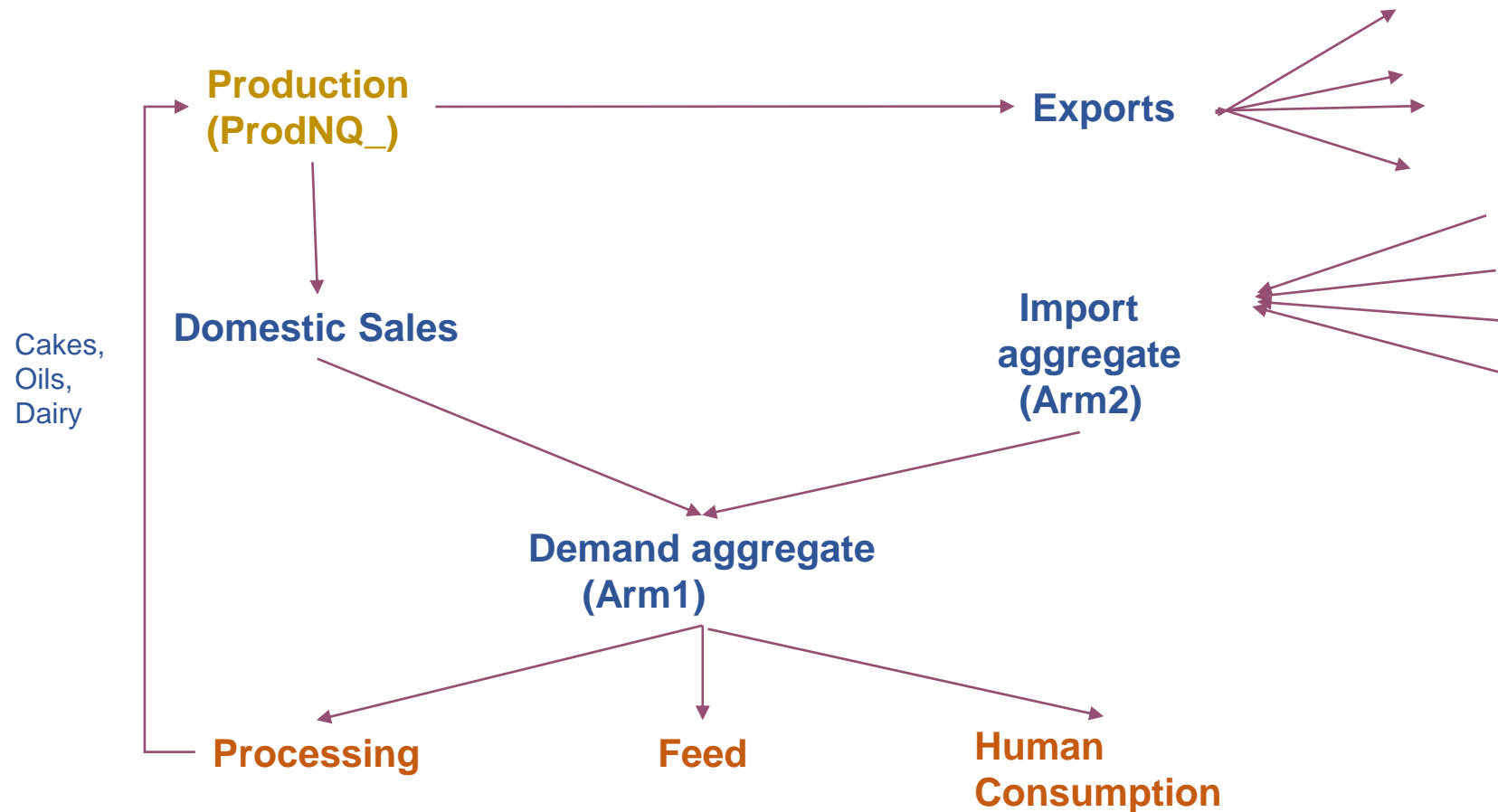
ρ = how easily consumers can substitute between domestically produced goods and imported goods (**elasticity of substitution = (σ=1/(1+ρ))**)

- A higher ρ (lower σ) implies less substitutability, indicating that consumers strongly differentiate between origins.
- A lower ρ (higher σ) means greater substitutability.

If, for example, the price of imports from r2 decreases relative to domestic goods in r1, the ratio M (representing imports) changes, leading to higher imports.

**GLE demand (X<sub>i,r</sub>):** Consumers in a region 'r' have an overall demand for a composite good 'i'.

# Quantity relations in market model



# Pros & Cons of Armington Approach

## Cons

- Few empirical estimations of the parameters  
=> substitution elasticities are set by a “rule-of-thumb”
- A zero stream in the calibrated points remains zero in all simulation runs
- Its applicability to simulate drastic shocks is limited
- Overestimated terms-of-trade effects, underestimated allocative efficiency gains from trade liberalization (Zhang, 2008)
- The sum of physical streams (domestic sales + imports) is not equal to the utility aggregate in simulations !!!  
(demand “quantities” are not longer tons, but a utility measurement ...)

## Pros

- Enables modelling “cross-hauling” in international trade (simultaneous import and export in the same trade relation)
- allows for calibrating the model to databases of international trade
- Can be calibrated to any price/quantity relations
- Appealing properties to model trade of heterogenous goods

# Exercise all

- Search `p_rhoArm1` in `gams/arm/market1.gms`

```

97 * ---- rho and elasticity of substitution parameter for Armington
98 *
99 * --- set substitution elasticities for Armington share
100 *   equation (drive reaction of trade flows resp.
101 *   domestic sales versus imports to price changes)
102 *
103 * p_rhoArm1 (RM,XX)      = 8.0;
104 * p_rhoArm2 (RM,XX)      = 10.0;
105 *
106 * -- sugar is homogeneous
107 *
108 * p_rhoArm1 (RM,"SUGA") = 12.0;
109 * p_rhoArm2 (RM,"SUGA") = 12.0;
110 *
111 * p_rhoArm1 (RM,"DDGS") = 2;
112 * p_rhoArm1 (RM,"DDGS") = 2;
113 *
114 * p_rhoArm1 (RM,XXBIOF) = 3.0;
115 * p_rhoArm2 (RM,XXBIOF) = 5.0;
116 *
117 *
118 * -- specific problems ...
119 *
120 * p_rhoArm1 (RM,"TAGR") $ RM_TO_RMTP (RM,"TUR") = 0.5;
121 * p_rhoArm1 (RM,"TABO") $ RM_TO_RMTP (RM,"TUR") = 0.5;
122 *
123 * -- huge yield shocks (in US) created trade related feasibility problems. Avoided with reduced substitutability for EU_WEST:

```

Search & Replace dialog box details:

- Search Term: `p_rhoArm1`
- Replace: (empty)
- Options:  Case Sensitivity,  Use Regex,  Whole Words
- File: This File
- Matches: 14 matches.

# Guided Exercise

- RMSSUP:
  - Level of supply/demand
  - Countries with a counterpart in the supply model (EU countries and a few other, Serbia, Montenegro, ...)
- RMS:
  - Level of supply/demand
  - Non-EU countries and country blocks (e.g. Brazil, the Middle East)
- RM:
  - Level of trade
  - EU\_WEST and EU\_EAST; flexible grouping for EU countries
- RMTP:
  - Level of Trade Policies (political units)
  - “EU” for the EU-27

# Exercise (guided): countries and commodities

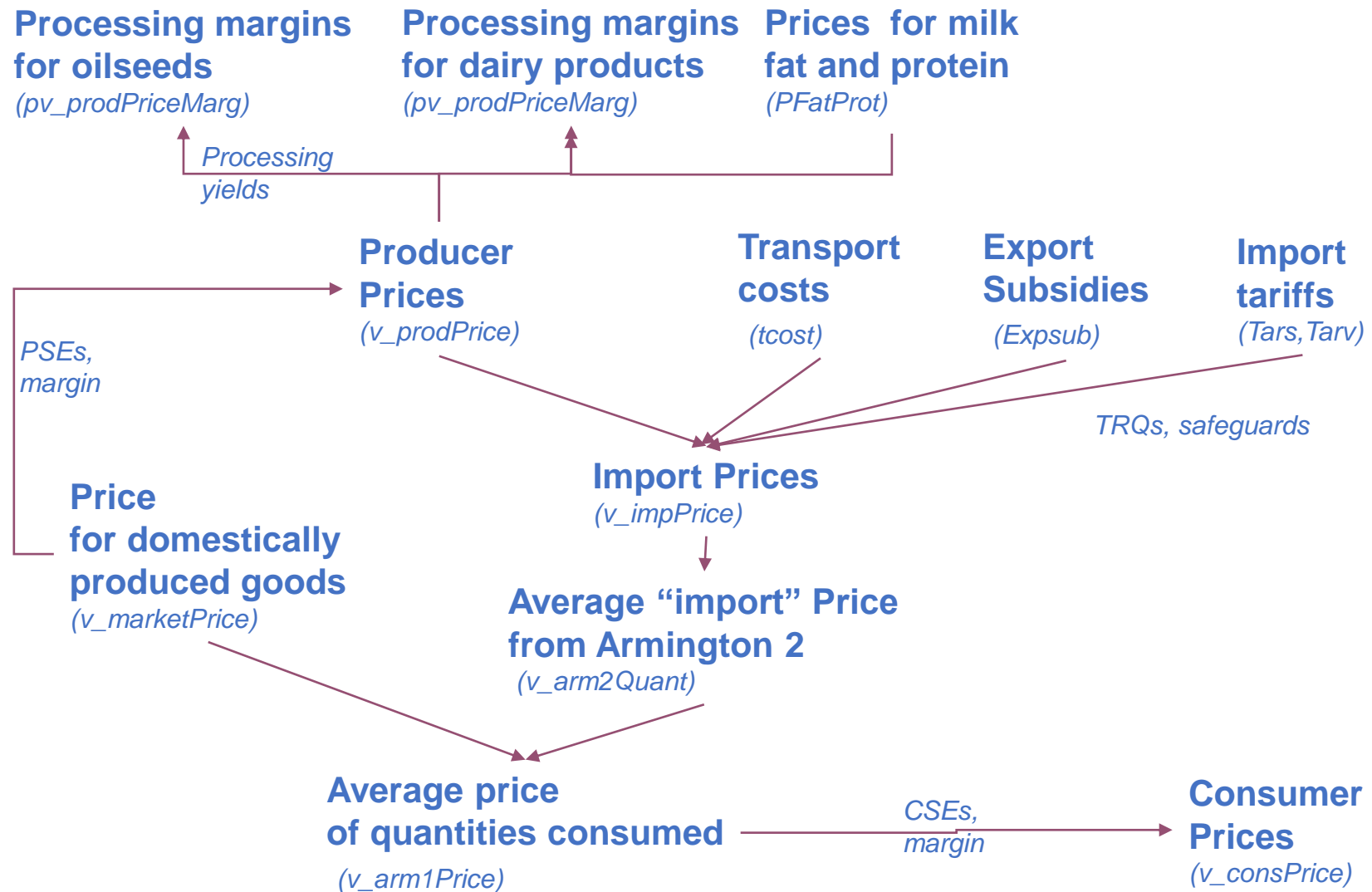
Open database of CAPRI.gdx with Studio and check which regional sets Ireland belongs to. The levels of the regional nest are stored in different GAMS sets and mapping sets: RMSSUP, RMS, RM, RM\_TO\_RMS, RM\_TO\_RMTP, RMS\_TO\_RM

- Nuts2 Code for Ireland:
- RMSSUP code for Ireland:
- RMS code for Ireland:
- RM aggregate that includes Ireland:
- RMTP aggregate that includes Ireland:

# 3. Balances

- Market balances (supply and utilization)
- Nutrient and energy balance:  $\text{sum}(\text{nutrient/energy content of feed}) = \text{sum}(\text{nutrient/energy requirement of animals})$
- Fats and protein balance for dairy products:  $\text{content in raw milk} = \text{sum}(\text{content in dairy products})$

# 4: Price relations in market model



# 5. Policy representation

- Custom duties (tariffs)
- Ad valorem (%) and specific (EUR/t) tariff rates
- Model-endogenous tariffs under Tariff Rate Quotas (tariff rate linked to imported volumes)
- Some EU-specific instruments: flexible levy, public intervention, export subsidies, entry-price system for fruits and vegetables

# Parameters and Variables in the Market Module

## Fixed parameters

- *Parameters in behavioural functions:*
  - Supply
  - Processing
  - Human consumption
  - Feed Use
- *Technical parameters:*
  - Crushing yields
  - Fat & protein content of milk products
- *Prices:*
  - Base year price producer
  - Marketing span for final products
- *Parameters in functions determining interventions and subsidized exports*

## Scenario parameters

- *Demand shifts:*
  - Population growth
  - GDP development
  - Changes in consumption pattern
- *Shifts in behavioural functions*
- *Exchange rates*
- *Policy instruments:*
  - Administrative prices
  - Maximal market interventions
  - Import Tariffs
  - Tariff Rate Quotas
  - Minimal import prices
  - Subsidised exports
  - Commitments
  - Non market PSEs
  - CSEs

## Endogenous Variables

- *Quantities:*
  - Supply
  - Processing
  - Human consumption
  - Feed Use
  - Intervention sales
  - Bilateral trade flows
- *Price elements:*
  - Market prices
  - Producer price
  - Consumer price
  - Processing margins
  - Import prices
  - Export subsidies
  - Tariffs



# What is a Multi-Commodity Model?

ⓘ Start presenting to display the poll results on this slide.



## The CAPRI Multi-Commodity Model (market model) is...

ⓘ Start presenting to display the poll results on this slide.



## The units of the market model are...

ⓘ Start presenting to display the poll results on this slide.



**Functional forms are often taken such that...**

ⓘ Start presenting to display the poll results on this slide.



# International trade is presented in CAPRI market model...

ⓘ Start presenting to display the poll results on this slide.

slido



**Exogenous scenario  
parameters in the market  
model are...**

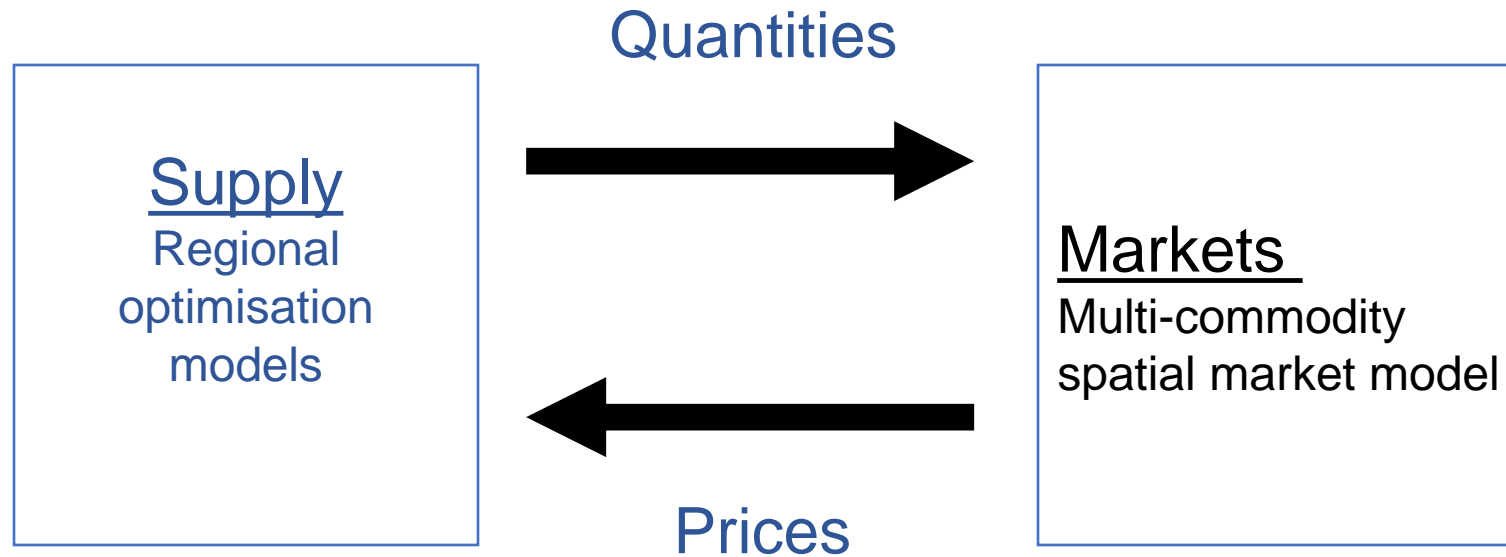
ⓘ Start presenting to display the poll results on this slide.



**Convergence is achieved  
between the supply model  
and the market model ...**

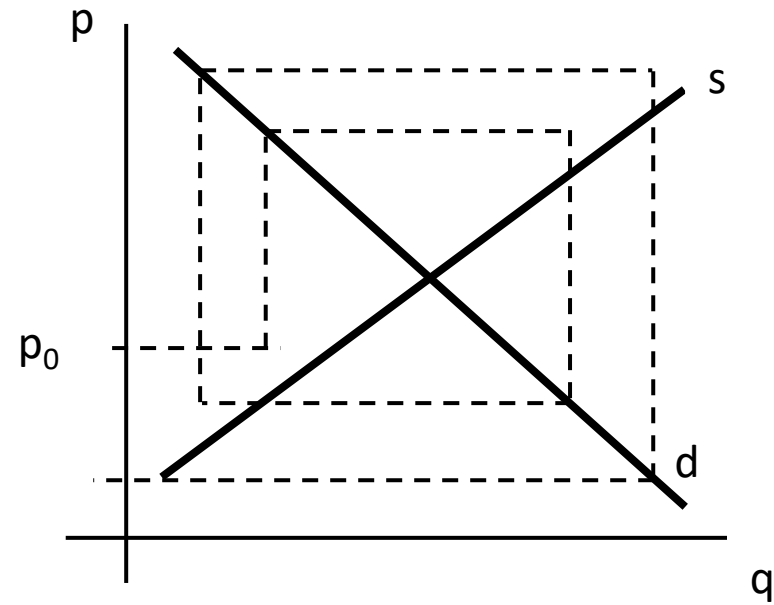
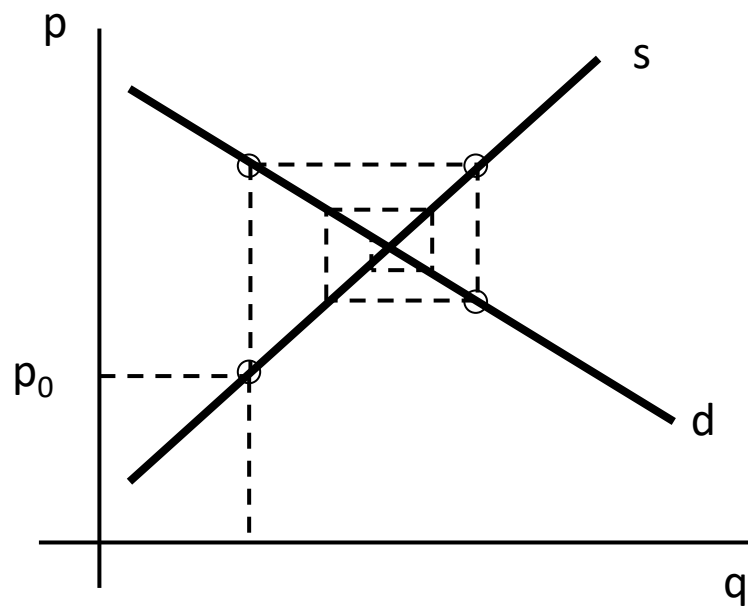
ⓘ Start presenting to display the poll results on this slide.

# Reminder – General Model Layout



Iterations → Comparative Static Equilibrium

# On convergence



# Conclusions

- If “demand elasticity”  $>$  “supply elasticity”, it will converge, otherwise not
  - CAPRI has to be solved iteratively
  - Elasticities are chosen based on economic criteria not to obtain convergence
- ⇒ **We will likely need some mechanism promote convergence in CAPRI**

# Different ways of promoting convergence in CAPRI

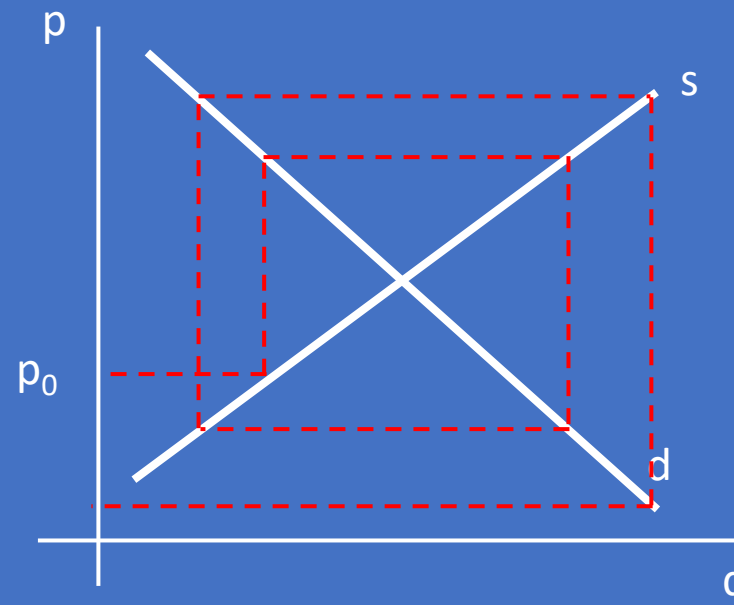
- Price expectation: Supply uses weighted average of prices in several previous step.
- Approximate supply functions used in market instead of fixed supply using sensitivity results from the supply models

# Approximation of supply functions

- The implicit supply function is unknown
  - Difficult to derive for CAPRI
  - Has non-differential points (corners) → difficult to solve together with market model
- Assume “any” simple supply function that approximates the supply model
- Calibrate the parameters in each step so that the supply response of last step is reproduced

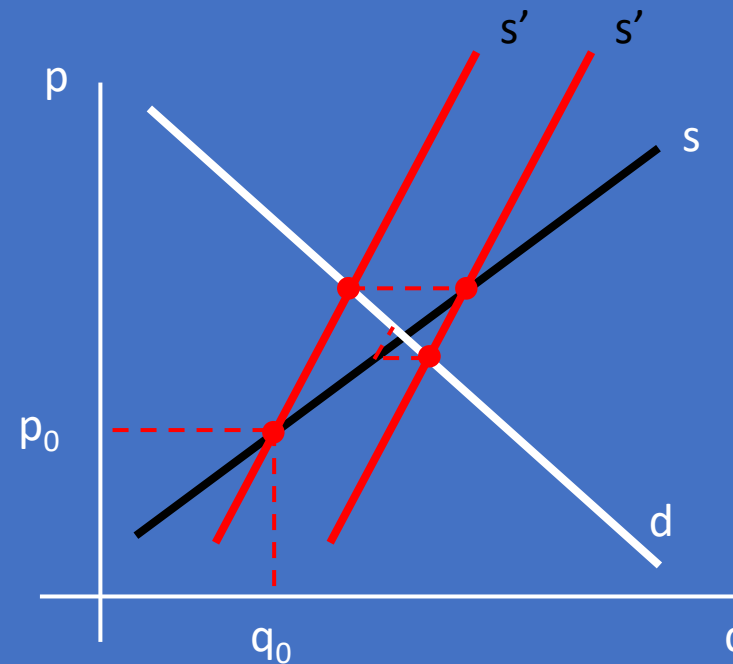
# Approximating supply

- Assume the “explosive situation” ...



# Approximating supply

- Supply function is unknown (supply is a black box)
- Assume any supply function
- Starting with some price, compute supply
- Calibrate the assumed supply function to that point
- Solve supply + demand simultaneously for new price
- Iterate...



# Technical implementation: CAPMOD Reminder

